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Diffused Light Interferometry for Measurement of Isopachics

by

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Abstract

A simplified optical method, based on interference phenomena of diffused light, is described for the determination of isopachic patterns.

Procedure consists in using a monochromatic diffused light beam to illuminate the model surface, If an optically flat glass plate, used as datum plane, is brought in contact with the model front surface an interference pattern is produced related to the thickness of the air-film between the datum plane and the surface of the model.

Interference patterns before and after loading of the specimen allow the determination of isopachics.

Curves of variation of interference fringes throughout the surface of the specimen, before their use for the determination of thickness variation, must be readjusted in convenient scale as well as for the elimination of superimposed simple interference patterns due to arbitrary inclination of the datum optical plane before and after loading. The method does not necessitate a complicated equipment. It is straightforward and very sensitive. It can be applied to transparent as well as to opaque models made of metal sheets. It can also be, used as a supplementary means to measure ε_z-strains in the birefringent coating method. The surface of the model though polished could not be obligatorily optically flat.

The method was applied to several illustrative examples with transparent and opaque models. The results obtained are in good agreement with those yielded by theoretical or other experimental solutions.

Introduction

Interference fringes between two beams of monochromatic light produced in suitable interferometric apparatus have long been used for refined measurement of thickness variation in terms of the wavelength of light in two-dimensional elasticity problems. In all these measurements thickness variation is determined in terms of a unit which is

half the wavelength of light used. This unit is of the order of one hundred thousandth of an inch. The precision of this order is more than satisfactory for any laboratory measurement of thickness.

Mesnager[1] was the first, in 1900, who used the phenomenon of interference of light for the direct measurement of thickness variation of models placed between two optically flat glass plates. Versatile lateral interferometer strain gages were constructed later by Vose[2] and Schaid[3]. Favre[4] used the Mach-Zehnder interferometer for point-wise measurements of the changes in the refractive indices on the two principal planes of models. Tank[5] modified Favre's method by introducing a quarter wave plate. This enabled him to measure directly the variation of the sum of principal stresses. Sinclair[6] and Bubb[7] described methods for obtaining isopachics all over the elastic field. The first used a Mach-Zehnder interferometer and the second a Michelson interferometer. Sinclair[8] simplified further Bubb's method by using lucite material presenting very low birefringence. Maris[9] and independently Tesar[10] measured the thickness variation of models by evaluating the interference fringe pattern produced by the air-film contained between an optical flat datum plane and the front surface of the model. Frocht[11] applied the same method in practical problems with opaque models which had their front surfaces polished optically flat. All these methods need for accurate results an extremely careful preparation of the specimen and a delicate technique of observation to eliminate misalignment of specimen and the optical flat.

On the other hand, Fabry[12] introduced a method for the separation of principal stresses by determining the interference fringes produced by the two half-silvered surfaces of the model. The method necessitated an almost absolute parallelism of the two surfaces of the model. He used glass specimens as models and he obtained a double system of fringes which could be separated only by the introduction of an analyser in the optical system. Dose and Landwehr[13] proposed a method using the system of interference fringes of a model, which was originally not optically flat, as a grating to produce the isopachic pattern by moiré effect. They applied their method to measure the flexure of plates[14]. The two flat surfaces of the model which were used as reflecting surfaces must be parallel. The method of Dose and Landwehr was applied by Drouven[15], Mesmer[16] and Post[17,18,19]. Post[20] has introduced a new versatile large field interferometer well adapted for measurement of minute variations ----------------------------

in optical paths through transparent models. Post's interferometer was modified by Frappier[21], who introduced a third independent half mirror, treated only on the one surface, instead of using two half mirrors on the same glass plate, and she obtained satisfactory results with isopachics. Pirard[22], in a comprehensive paper, studied extensively the phenomenon of production of moiré fringes and applied the method of Dose and Landwehr for the determination of isopachics. In a more recent paper[23] he showed the possibilities of the same method for an effective solution of hyperstatic problems. The method described in the paper is shown to be the simplest of all previous methods giving satisfactory results without the need of elaborate instruments and delicate techniques for the measurement of isopachics.

Experimental Procedure

Optical interference yields an effective means for the study of the topography of specularly reflecting surfaces with a simple and inexpensive equipment. The procedure involves estimation of deviation of a surface from planeness by observation of the interference pattern formed by the air-film included between the reflecting surface of the model and a datum surface of an optical flat placed in contact with it. A beam of diffused monochromatic sodium yellow light illuminates the surface of the model by a small angle of incidence. A photographic camera is placed with its optical axis coinciding with the reflection direction of the diffused light. If the datum plate made of an optically flat glass plate covers the reflecting surface of the model an interference pattern is shown in the ground glass of the camera. Fig. 1. shows the optical path of the diffused light. The datum plate rests on the polished surface of the model and rides on the high points of the surface held by elastic strings. These strings apply small forces sufficient to keep the optical plate in place without introducing any flexure on the plate or applying any transversal load on the model. The moment produced on the model from the eccentrically placed optical plate must be counterbalanced by another glass plate of the same dimensions and weight, which is placed on the opposite side of the model held by the same strings. Between this second glass plate and the model a black piece of paper is put which increases considerably the contrast of the interference fringes shown in the yellow light.

In a two-dimensional stress field there is, in general, a plane of symmetry which is the only one remaining plane under any load producing a plane-elastic field. To this plane it is related the inherent plane of symmetry of the model. The datum surface may not be parallel to the inherent plane of symmetry of the model due to the eccentrically supported glass plate and especially due to small differences in elevations of the high points in the model itself. Thus, an interference pattern of the surface of the model is shown corresponding to a fictitious loading producing thickness variations due to differences in path between the datum plane and the unloaded model surface. The unloaded interference pattern is photographed before any loading, giving the so-called original pattern.

The model was placed in a straining frame and a convenient amount of loading was applied to the model. Because of the small size of the unit of measure used, i.e. half the wavelength of light, the applied load must be small enough to produce a sufficient number of fringes. Exceeding loads produce great number of interference fringes hard to be photographed and analysed. The new pattern, which will be called hereafter the loaded pattern, contains, besides the original pattern due to variation of thickness of the model, another pattern due to eventual rotation of the datum plane. Indeed, since the datum plane is riding on the high points of the model and since there is, in general, a variation of the elevations of the high points, due to change of thickness, some rotation of the datum plane may be expected. Flexure of the datum plane or the model may be excluded by a proper fixing of the optical glass plate and its counterbalancing plate on the model. It is preferable to take first the photograph of the loaded pattern and then to unload without changing the position of the model and take the photograph of the original pattern. This procedure has the advantage to eliminate any error introduced by changing the relative position of the model, the light source and the camera.

The first operation to do after the taking of the photographs of the original and the loaded pattern is to establish a correct correspondence between the two patterns. This means that it is necessary to know in what direction, normal to fringes, their order increases or decreases. In fact, it suffices to adopt a concordant direction in all points of both patterns without caring of the true increase or decrease of fringes. This procedure is very simple, provided care is

given to recognize properly the tops, valleys and necks of the topographic representation of patterns. In the second pattern of the loaded configuration it is necessary not only to know the direction of variation of fringes but also the chosen direction to agree with the already established in the original pattern. Then, the correspondence of both patterns is correct and they are called concordant. For this it suffices to know the direction of isopachics in some place of the model. This place can be any free boundary, where the sum of principal stresses coincides with their difference. The probable shape of isopachics in the vicinity of these boundaries can be estimated easily. If the direction of increase is already established in the one pattern (say, the original pattern), the direction of increase of fringes in the other pattern can be established by considering that any isopachic curve results from subtraction of the two patterns. Therefore, the difference of order between the two patterns along any isopachic must be constant. From this fact the direction of increase in the second pattern is determined (see for example Fig. 2).

For the separation of principal stresses along any line transversing the model we trace first their difference from the corresponding isochromatic pattern. For the determination of the corresponding isopachic pattern we trace the diagram of thickness variation of the model before loading. This diagram is traced with an arbitrary scale because the thickness variation corresponding to one interference fringe is unknown, and with an arbitrary shifting from zero level because the absolute order of fringes is also unknown. The diagram of the loaded configuration of the model is traced in the same manner with the same scale of fringes used previously and respecting also the already adopted concordant direction of fringe variation. The difference of the two diagrams yields the diagram of thickness variation provided that the datum plane does no rotate during loading and it remains parallel to its initial position of the unloaded model.

Indeed, if we trace the curves of variation of interference fringe orders along any cross section of the model with an arbitrary scale q inches per fringe order we obtain from the two patterns the elevations $z_{1,2}$ of the unloaded and loaded case respectively

$$z_1 = q\left[f_1(x) + c_1\right]$$
$$z_2 = q\left[f_2(x) + c_2\right]$$

where $f_1(x)$ denotes the variation of fringe orders for the original pattern and $f_2(x)$ denotes the variation for the loaded pattern. The curve of the differences will have as elevations

$$z = z_2 - z_1 = q \left[(f_2 - f_1) + c_2 - c_1 \right] \qquad (1)$$

The true values of ordinates of the corresponding isopachic curve are expressed by the following relation:

$$(\sigma_1 + \sigma_2) = k' (f_2 - f_1) \qquad (2)$$

where the coefficient k' contains the characteristic constants of the interference model i.e. the modulus of elasticity, Poisson's ratio and its thickness.

If the isopachic curves would be refered to the scale p of the isochromatic related to the corresponding photoelastic model then

$$Z = pk (f_2 - f_1) \qquad (3)$$

where the coefficient k takes care of the influence of the different elastic constants of the photoelastic material. But, from relation (1) we have

$$(f_2 - f_1) = \frac{z}{q} + (c_1 - c_2)$$

then

$$Z = \frac{pk}{q} z + pk (c_1 - c_2)$$

or

$$Z = \varphi z + c \qquad (4)$$

For the determination of the Z-curve from the z-curve, which results from the difference of the two patterns, it is necessary to evaluate the coefficient φ and the constant c. The values of these parameters can be determined from the known values of Z in two points of the section. If A and B are the known points it can be proved that

$$\varphi = \frac{Z_B - Z_A}{z_B - z_A} \quad (5) \quad \text{and} \quad c = \frac{Z_A z_B - Z_B z_A}{z_B - z_A} \qquad (6)$$

The elevation of a generic point of the section will then be

$$Z = Z_A + \varphi (z - z_A) \qquad (7)$$

where φ is determined from relation (5).

It may be observed that the values of z_A and z_B must not be equal. If $z_A = z_B$ then the value of φ is undetermined. The coefficient

φ contains the values of the two scales p and q of the isochromatics and interference fringes respectively the ratio $\frac{P}{p}$, of the two applied loads during the interferometric and photo-elastic experiment and the constants contained in the coefficient k. For constant values of all these parameters the values of φ must be the same all over the model. Therefore, in all sections of the same photoelastic and interferometric experiment the coefficient φ must remain constant.

In the case when the datum plane is rotated during the loading of the model the elevations $z_{1,2}$ of the unloaded and loaded model will be respectively

$$z_1 = q \; [f_1(x) + c_1]$$
$$z_2 = q \; [f_2(x) + c_2x + c_3]$$

where the constant c_2 expresses the projection of the angular coefficient of rotation of datum plane on a perpendicular plane to the model, passing through the considered section.

$$z = z_2 - z_1 = q \; [f_2 - f_1 + c_2x + c_3 - c_1] \tag{8}$$

On the other hand, we have found that

$$Z = pk \; (f_2 - f_1) \tag{9}$$

then
$$Z = \frac{pkz}{q} - pkc_2x + pk(c_1 - c_3)$$

or
$$Z = \varphi z - C_1x + C \tag{10}$$

While relation (4) is valid for any kind of section made on the model, relation (10) is valid only for linear sections. In the case of curved sections the values of z are functions of the distances lx along the section while the quantities x in the term C_1x are the projections of the distances lx on the closing line of the non-linear section. Then, the coefficient C_1 is constant all over the section and expresses the projection of the angular coefficient of rotation of the datum plane on a plane passing through the closing line of the section, If the quantities x in the term C_1x must coincide with the distances lx along the non-linear section, then the coefficient C_1 must vary from place to place and it must be equal to the projection of the angular coefficient of rotation of datum plane on a plane tangent to the section at the considered point.

For the evaluation of the three coefficients φ, C_1 and C in Eq(10) we need to know the values of Z at three different points on the section. If A,B and C are these three points then

$$Z_A = \varphi z_A - C_1 x_A + C$$

$$Z_B = \varphi z_B - C_1 x_B + C$$

$$Z_C = \varphi z_C - C_1 x_C + C$$

Solving these three relations we obtain

$$\varphi = \frac{(Z_B - Z_A)(x_C - x_A) - (Z_C - Z_A)(x_B - x_A)}{(z_B - z_A)(x_C - x_A) - (z_C - z_A)(x_B - x_A)} \tag{11}$$

$$C_1 = - \frac{(Z_B - Z_A) - \varphi(z_B - z_A)}{x_B - x_A} \tag{12}$$

$$C = Z_A - \varphi z_A + C_1 x_A \tag{13}$$

In the common cases of symmetrical sections we can select two points for which we have $Z_A = Z_B$ and relations (11) to (13) become

$$\varphi = \frac{(Z_C - Z_A)}{(z_C - z_A) - (z_B - z_A)\frac{(x_C - x_A)}{(x_B - x_A)}} \tag{14}$$

$$C_1 = \varphi \frac{z_B - z_A}{x_B - x_A} \tag{15}$$

$$C = Z_A - \varphi z_A + C_1 x_A \tag{16}$$

If we take assymmetrical points A and D the extremities of the section, for which we have $x_A = 0$ $x_B = 1$ $z_B - z_A = \Delta z_o$

$$\varphi = \frac{(Z_C - Z_A)}{(z_C - z_A) - \frac{\Delta z_o}{l} x_C} \tag{17}$$

$$C_1 = \varphi \frac{\Delta z_o}{l} \tag{18}$$

$$C = Z_A - \varphi z_A \tag{19}$$

The elevation of a generic point of a symmetrical section, for which we know the values, Z_A, Z_B and the value Z_C of α third point, is given by relation

$$Z = Z_A + \varphi\left(z - z_A - \Delta z_o \frac{x}{l}\right) \qquad (20)$$

where the coefficient φ is determined from data obtained from the third point C.
From relation (10) it can be deduced again that the coefficient φ is constant for any section of the model.

For the determination of the coefficient φ use may be made of the correspondence of the isochromatic and the interference pattern along free boundaries. Indeed, along any free boundary the function Z is known. By making the curves of isochromatics and interference fringes to coindide along any free boundary it can be deduced easily the value of the coefficient φ.

The procedure for the determination of the other constants in problems presenting a symmetry can be further simplified as follows:

a) The tracing of interference curves along any free boundary from the original and the loaded pattern yields the curve of differences z. This curve can be related directly to the corresponding isochromatic curve and yields the value of coefficient φ all over the field of the model.

b) Any other linear section at the interior of the model parallel to an axis of symmetry yields a curve of differences z for which it is valid that the equidistant points from the middle point of section have elevations Z which must be equal. If we trace the closing line of the two extremities of the section, which ends at two symmetric points of the boundary, then the intercept of any pair of equidistant points must be parallel to the closing line. Therefore if we rotate the whole figure to make horizontal the closing line we can eliminate the influence of rotation of the datum plane during loading. This corresponds to a subtraction of the term C_1x in relation (10) and the evaluation of true elevations Z degenerates to the simple case without rotation of datum plane. In that case relations (4), (5), (6) and (7) are valid and the constant C can easily be determined for each section.

Indeed, formulas (4), (5), (6) and (7), valid for a parallel translation of the datum plane, can be used in the general case provided

that instead of the elevations z we take the values $z' = z - \Delta z_o \frac{x}{l}$, that is by making horizontal the closing line of the curve of differences z. Then, for the second extremity B the elevation will be $z'_B = (z_B - \Delta z_o) = z_A$.

It can then be proved that the coefficients φ and C are expressed as follows :

$$\varphi = \frac{Z_C - Z_A}{z'_C - z_A} \tag{21}$$

$$C = \frac{Z_A z'_C - Z_C z_A}{z'_C - z_A} \tag{22}$$

$$Z = \varphi z' + C \tag{23}$$

or

$$Z = Z_A + \varphi(z' - z_A) \tag{24}$$

In the case of an non-symmetric section the general formulas (8), (9), (10) and (11) must be used for the evaluation of constants. Again, the value φ is constant all over the model and it can be determined at a free boundary.

Illustrative Examples

For the evaluation of the method developed it will be applied to two different problems. The first is that of a strip containing two semi-circular grooves subjected to pure tension. The model was made of plexiglas and its dimensions were as follows: thickness t=10mm, width of strip b=32mm and the diameter of the grooves D=16mm.

The model was first loaded to a convenient load so that the interference loaded pattern presented a sufficient number of fringes. The loadedpattern was photographed after elapsing a time distance for the creep of the model. Then, the original pattern was photographed by unloading the model and living it for a sufficient time to recover. Care also was taken not to let turn the model during unloading.

Figs (3 a,b) show the original and loaded patterns of the model. From these interference patterns in relation to the corresponding isochromatic pattern the sum of principal stresses was evaluated on the boundaries of semi-circular notches and along the axes of symmetry. The complete coincidence of the calculated values with

other theoretical and photoelastic solutions shows the accuracy of the method.

Figs (4), (5), (6) and (7) show the curves of elevations of the unloaded and loaded case as well as their differences along the boundaries of semi-circular grooves and the principal axes of the model and figs. (8) and (9) present the graphs of calculated values of the sum of principal stresses along the same lines.

As a second illustrative example was used the case of a circular ring subjected in compression.

The model was made of plexiglas and its dimensions were: thickness t=10mm. Outer diameter D=80mm. Inner Diameter d=40mm. Figs (10), (11), (12) and (13) show the two interference patterns of the ring (original and loaded) as well as the evaluation of the sum of principal stresses along the inner boundary and the cross section of the ring perpendicular to the axis of application of load. Again, the comparison of determined values of the sum of principal stresses with those obtained by Photoelasticity[11)] proves the accuracy of the method.

The same model was made from steel in order to show the possibility of application of the method to opaque materials. The one surface of the model was ground and polished to a fine emery. Then, this surface was covered with a very thin layer of a cold-setting epoxy resin (Epoxy Resin Epon 828) in which was added 8 percent of hardener (CIBA hardener 951) and a black colour pigment. The resin was cast between the metallic ring and the polished glass plate which was used later as a datum plane. The glass plate was covered with a thin layer of Dow Corning R-671 mold release agent for the easier separation from the epoxy resin layer. The model was left to set up and cure for a week at room temperature and then tested in yellow diffused light. The epoxy resin thin layer was added to give a better contrast to the produced interference fringes. Figs 14(a,b) show the original and loaded interference patterns of the metallic specimen.

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Captions of Figures

Fig. 1. Schematic diagram of diffused light interference apparatus.

Fig. 2. Interference fringes of loaded (numbers 1,2,3 etc.) and original patterns (numbers I,II,III etc.) and determination of their concordance by tracing the probable isopachic 0-0.

Fig. 3. The original (a) and loaded (b) interference patterns of a strip sontaining two semi-circular grooves-loaded in tension.

Fig. 4. The original and loaded elevation curves and their difference at the rim of semi-circular groove ACA of the strip.

Fig. 5. The original and loaded elevation curves and their difference at the rim of the semi-circular groove A'C'A' of the strip.

Fig. 6. The original and loaded elevation curves and their difference along the minimum section of the strip.

Fig. 7. The original and loaded elevation curves and their difference along the longitudinal axis of the strip.

Fig. 8. The variation of isopachics at the rims of the semi-circular grooves of the strip.

Fig. 9. The variation of isopachics along the minimum section and the longitudinal axis of the strip.

Fig. 10. The original (a) and loaded (b) interference patterns of a circular ring loaded in diametral compression.

Fig. 11. The original and loaded elevation curves and their difference at the interior boundary of the ring.

Fig. 12. The original and loaded elevation curves and their difference at the central horizontal section of the ring.

Fig. 13. The variation of isopachics at the interior circular boundary and the central horizontal section of the ring.

Fig. 14. The original (a) and loaded (b) interference patterns of a circular ring made of steel and subjected in diametral compression.

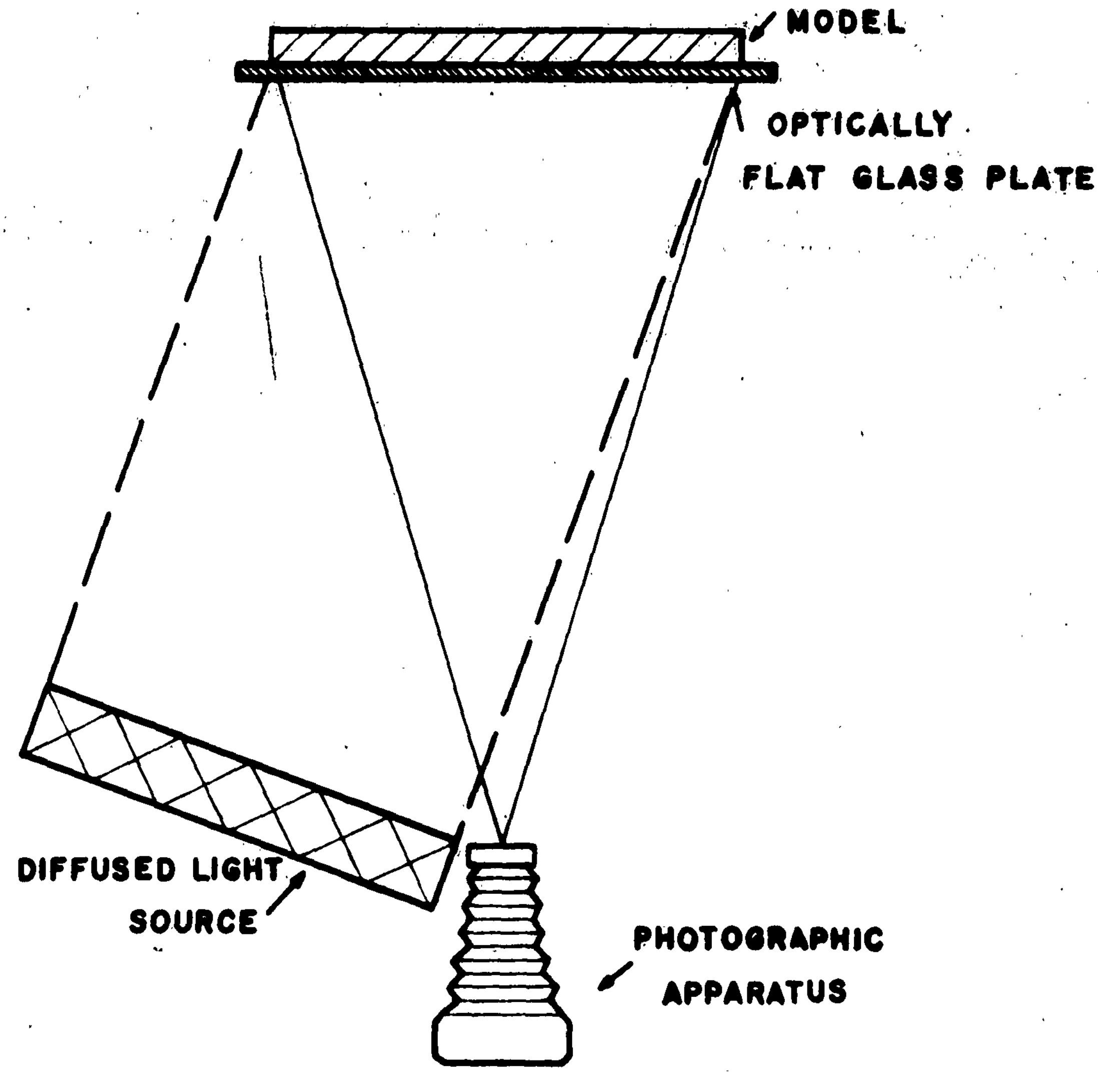

MODEL
OPTICALLY
FLAT GLASS PLATE
DIFFUSED LIGHT
SOURCE
PHOTOGRAPHIC
APPARATUS

FIG. I

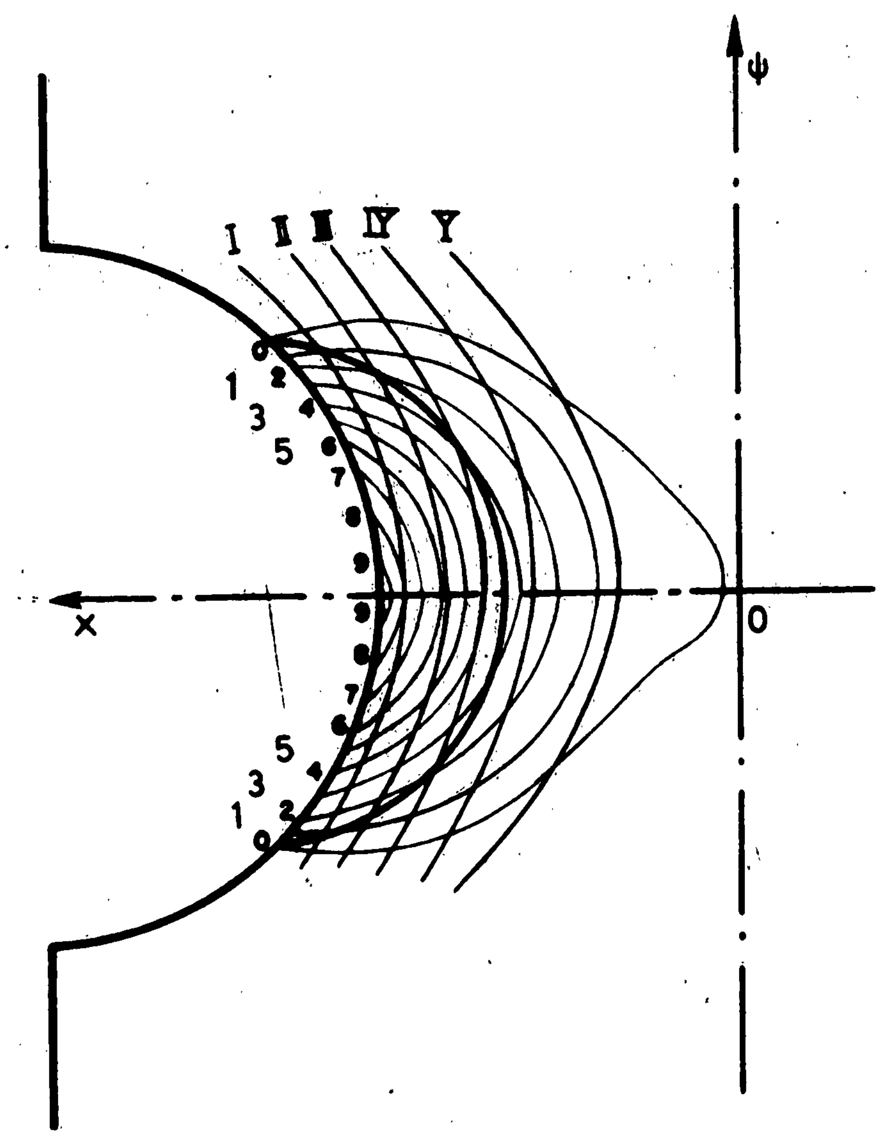


FIG. 2

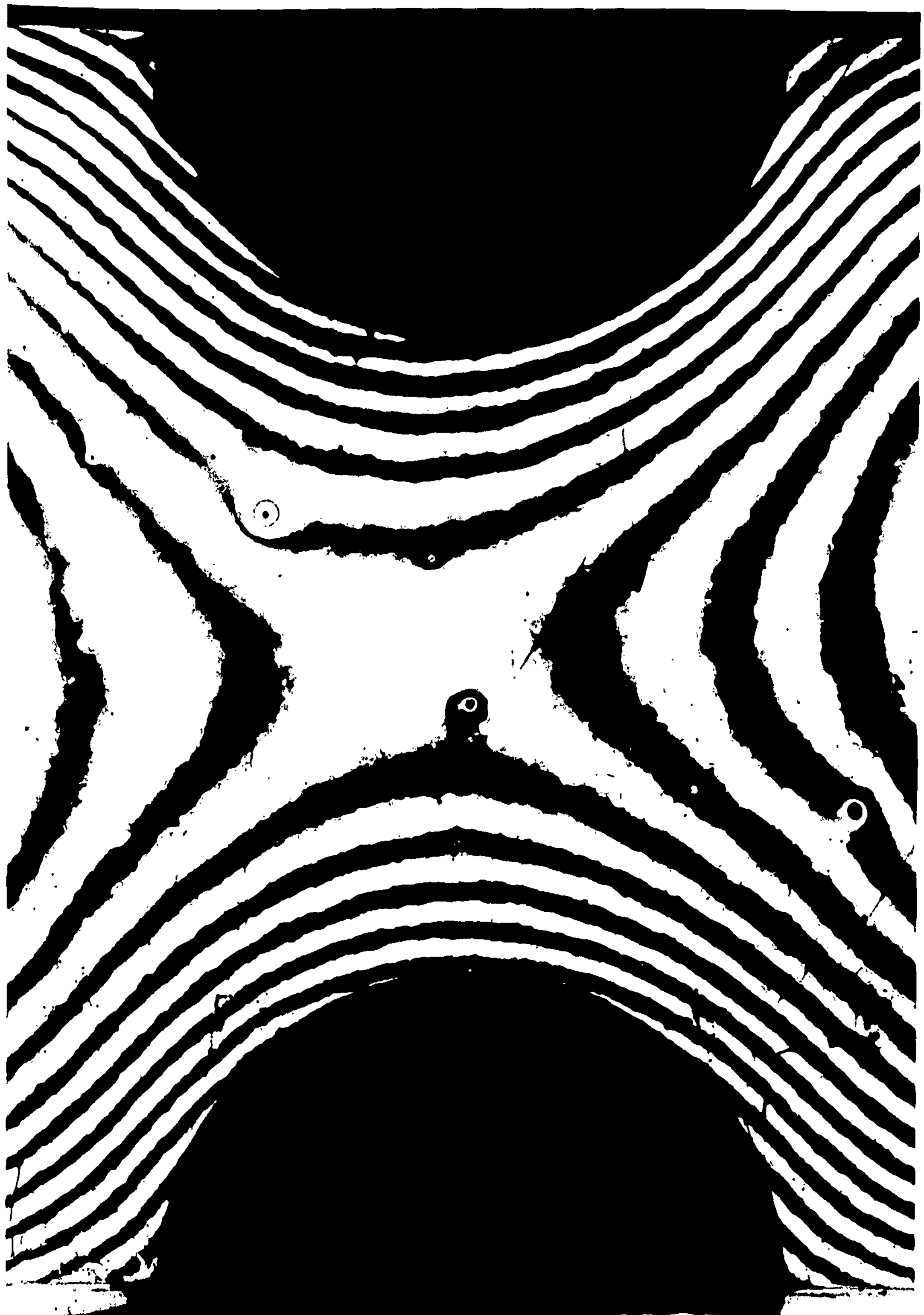


FiG 3(a)

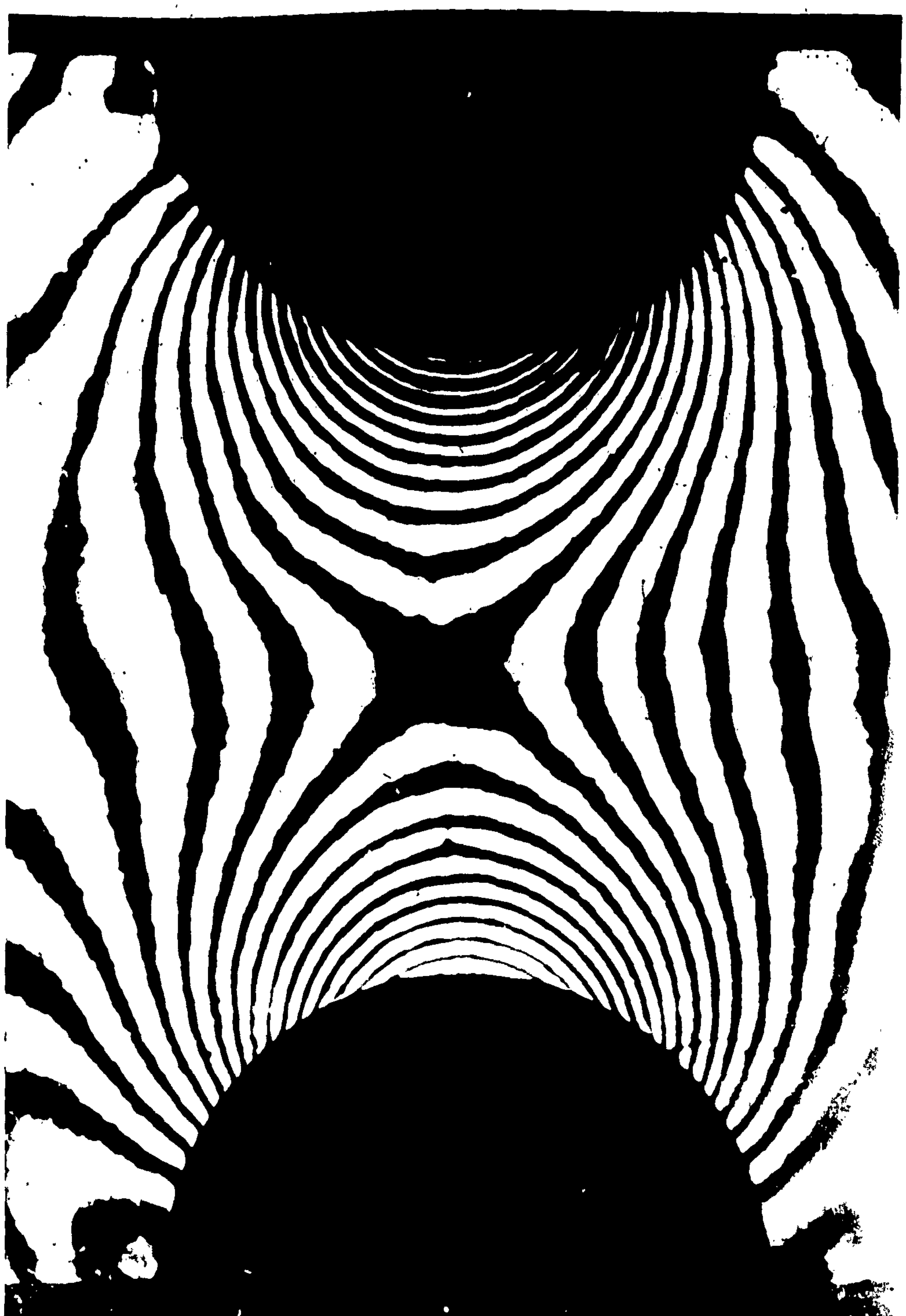
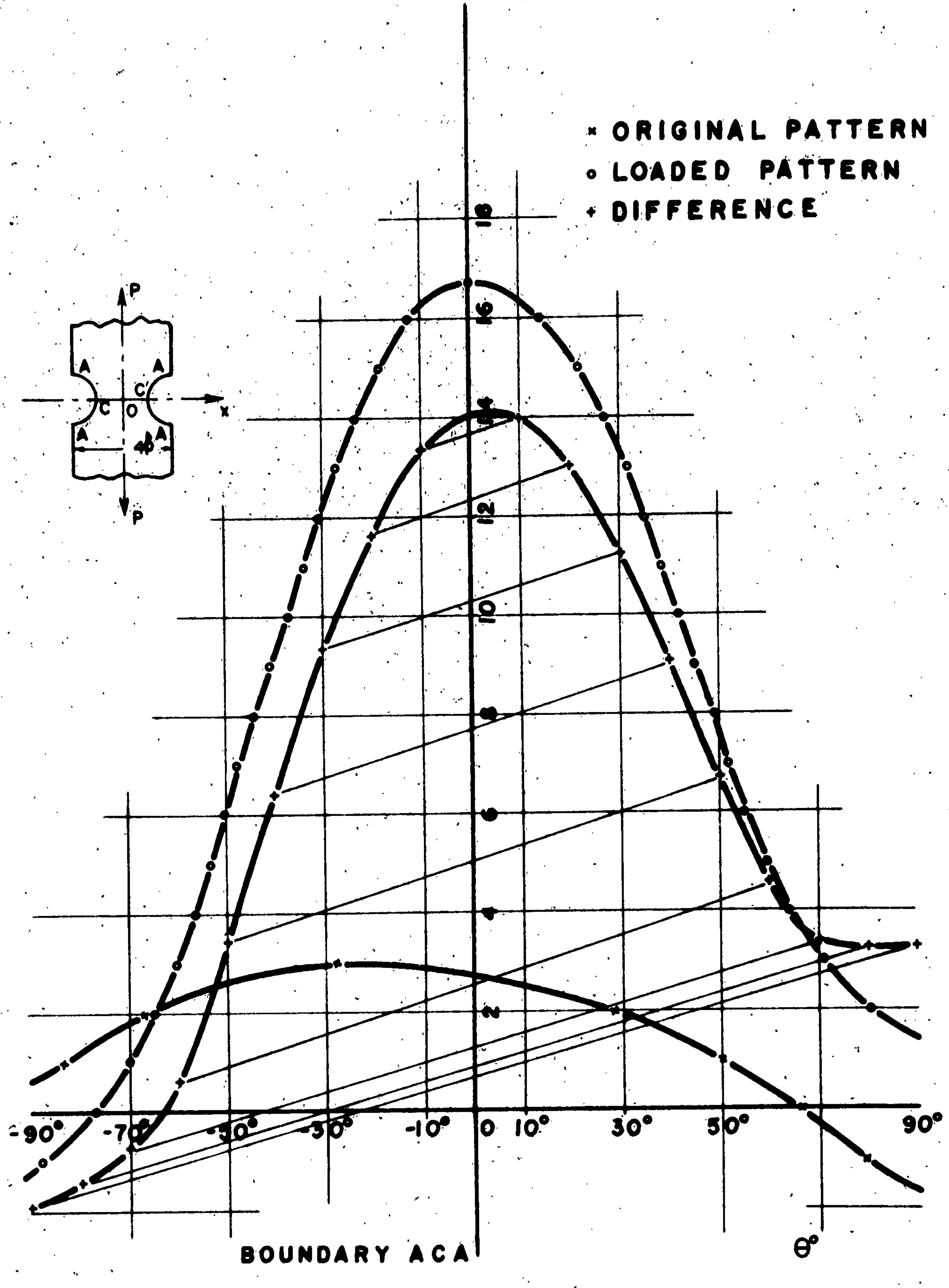


Fig 3(b)



× ORIGINAL PATTERN
○ LOADED PATTERN
+ DIFFERENCE
P
A
C
O
x
40
18
16
14
12
10
8
6
4
2
-90°
-70°
-50°
-30°
-10°
0
10°
30°
50°
90°
BOUNDARY ACA
θ°

FIG. 4

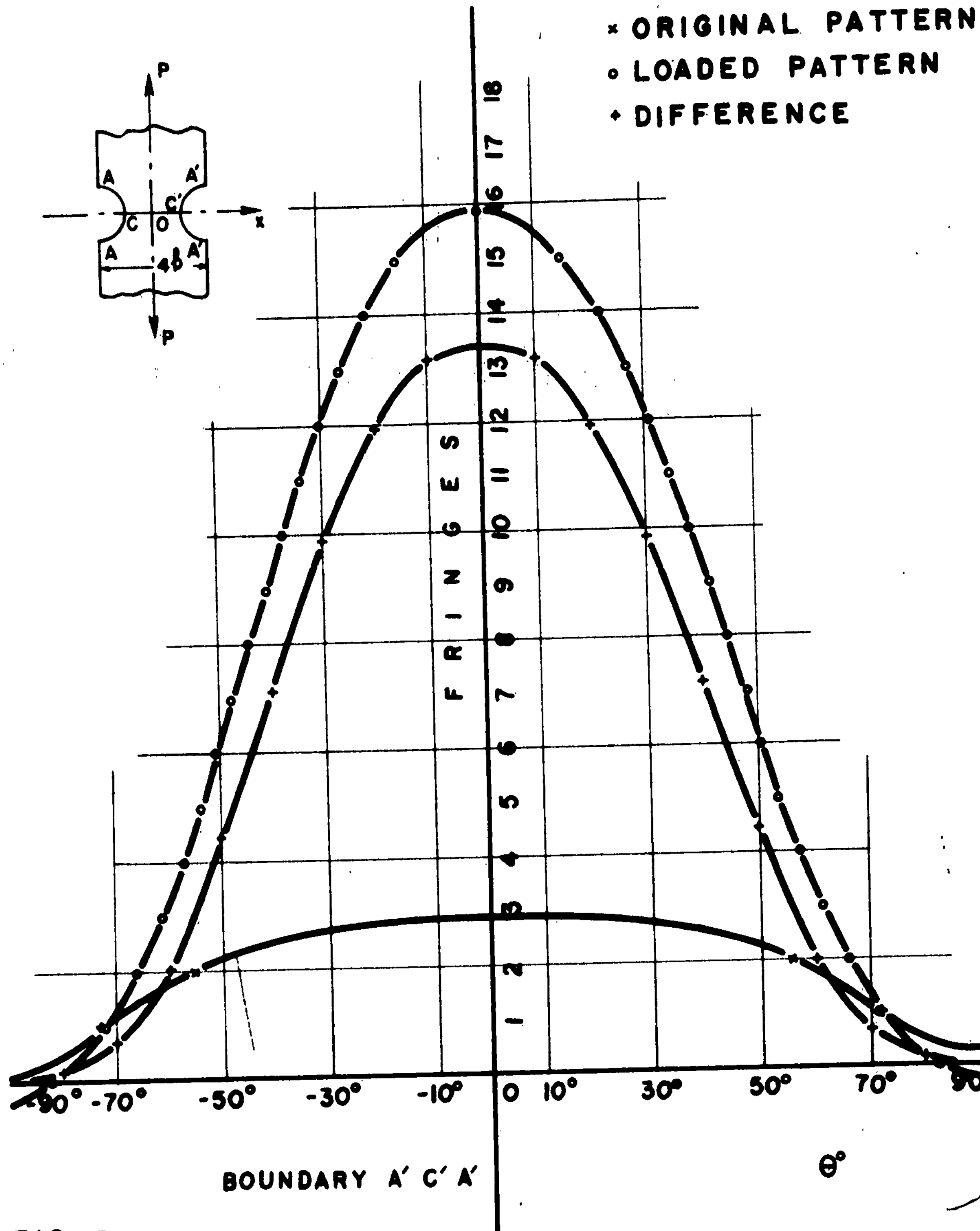
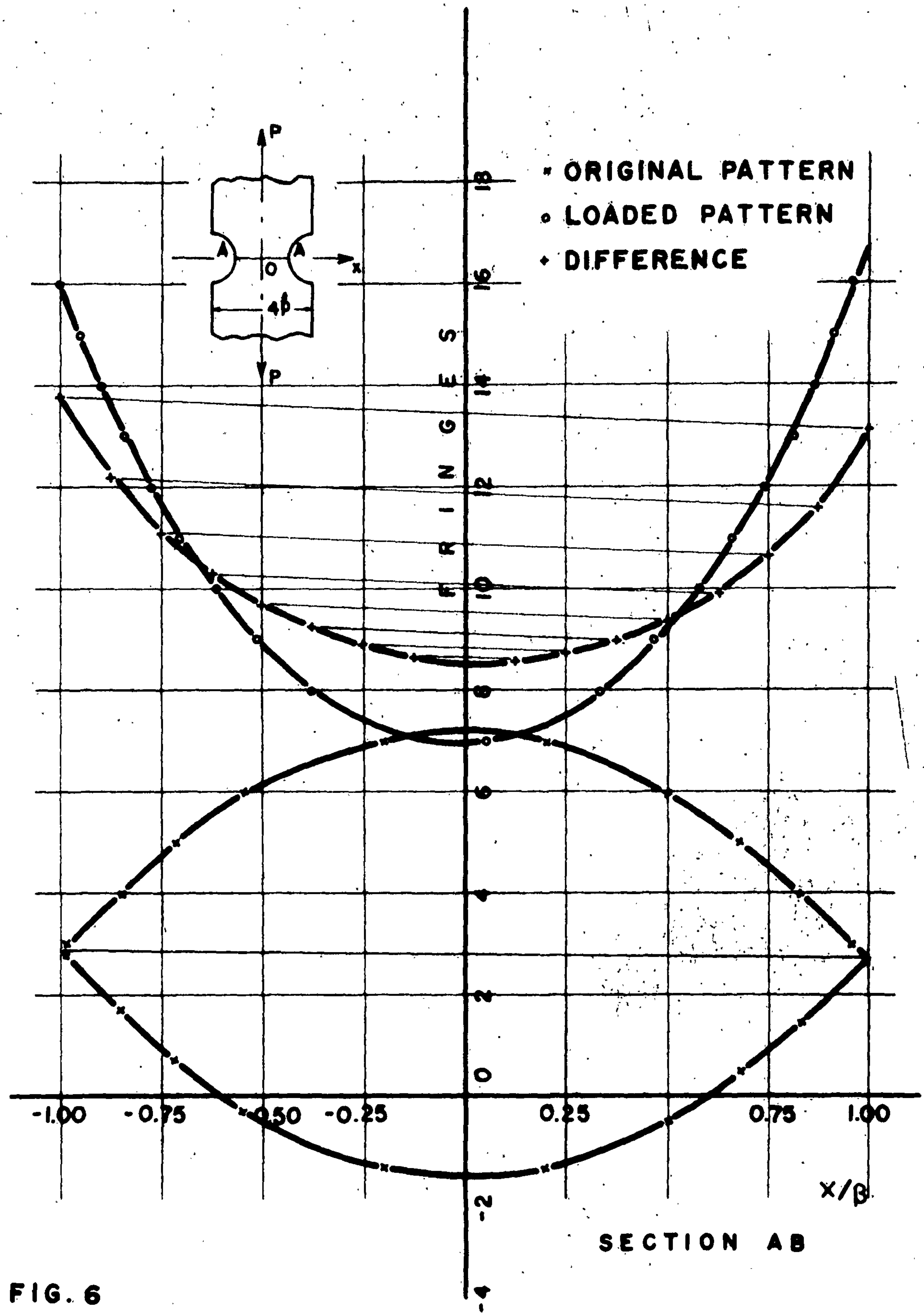
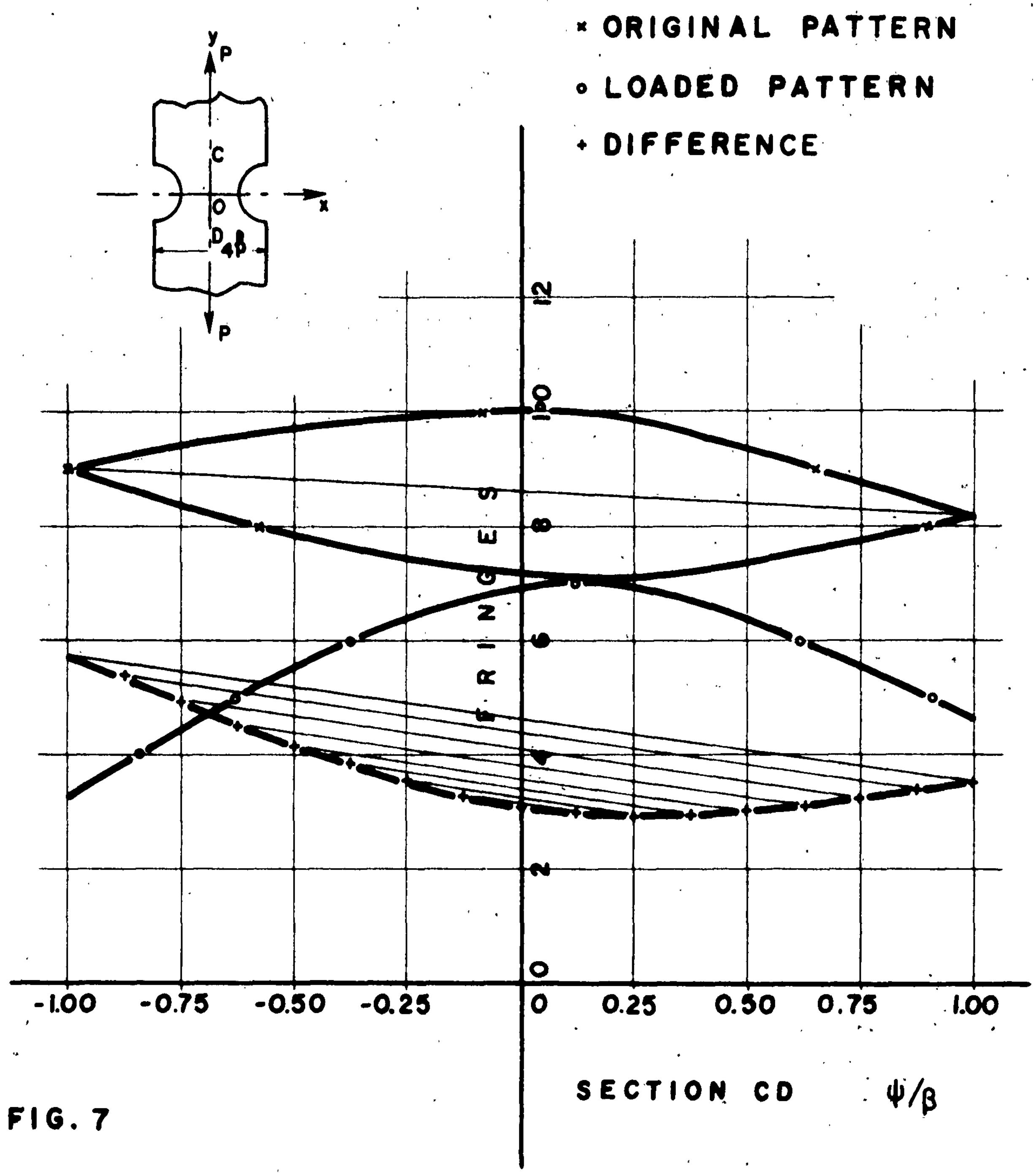

× ORIGINAL PATTERN
○ LOADED PATTERN
+ DIFFERENCE
P
A
A'
C
C'
O
x
4b
P
FRINGES
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
-90°
-70°
-50°
-30°
-10°
0
10°
30°
50°
70°
90°
BOUNDARY A' C' A'
θ°

FIG. 5



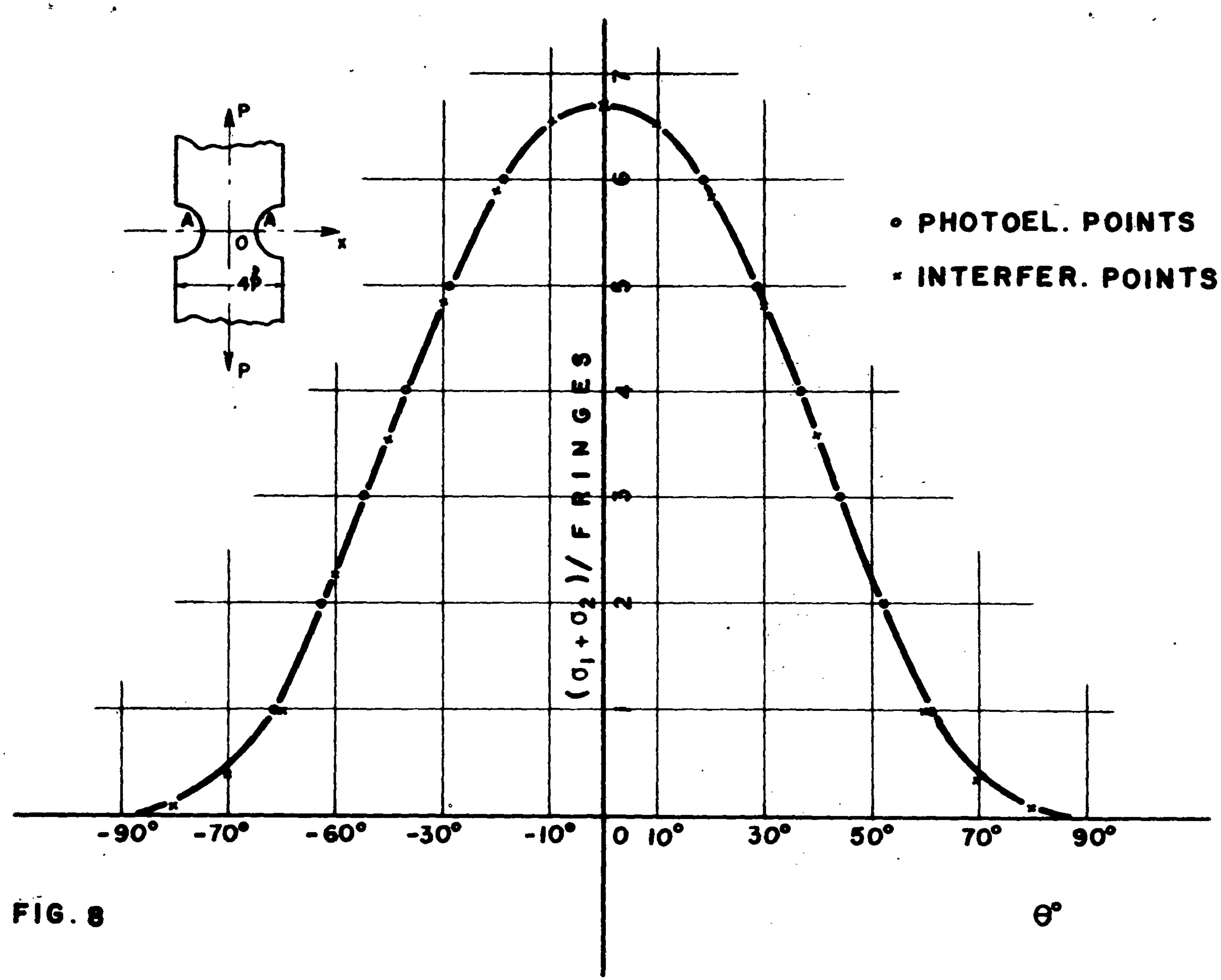
ORIGINAL PATTERN
LOADED PATTERN
DIFFERENCE
P
A
O
x
4β
FRINGES
18
16
14
12
10
8
6
4
2
0
-2
-4
-1.00
-0.75
-0.50
-0.25
0.25
0.75
1.00
X/β
SECTION AB

FIG. 6



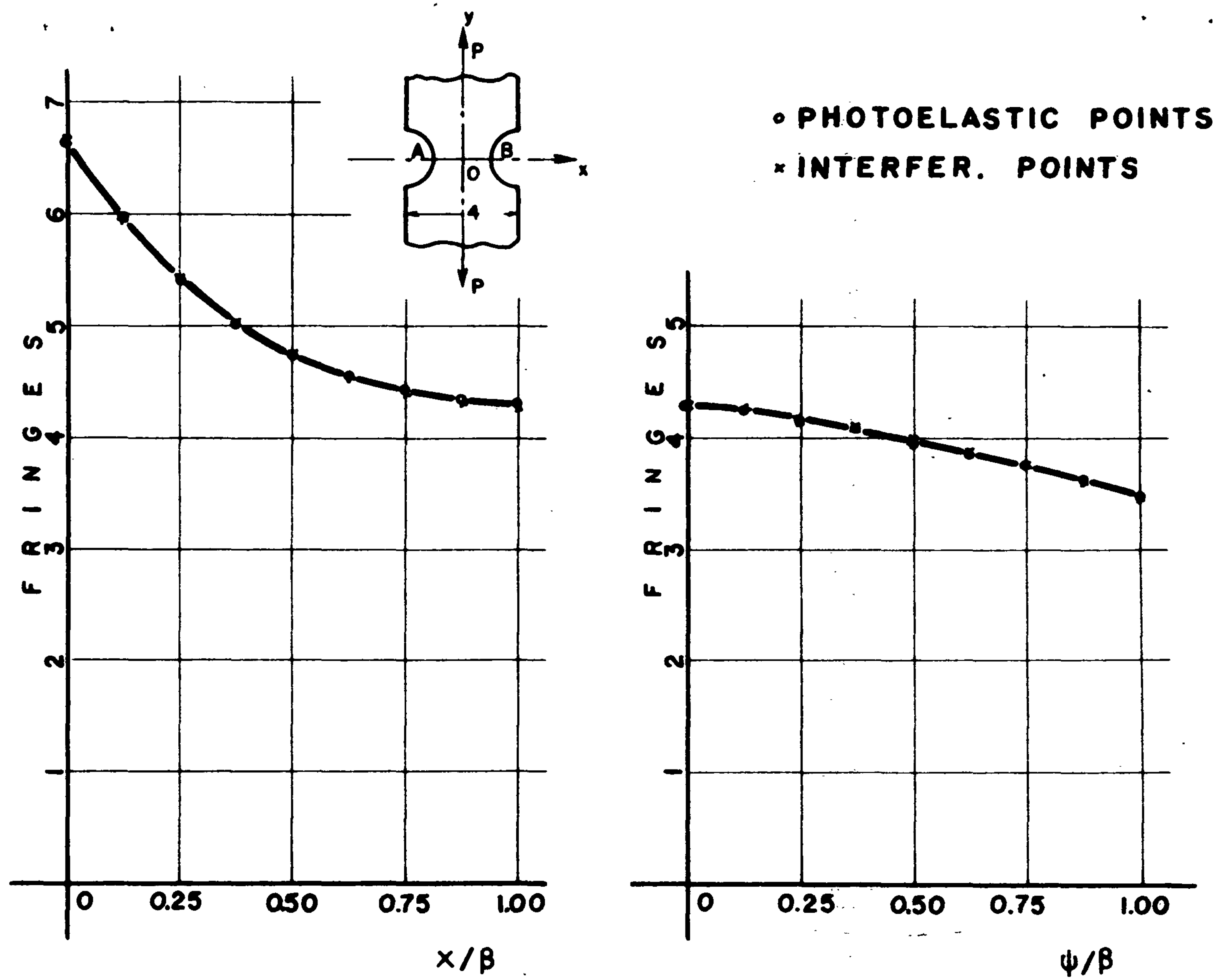
× ORIGINAL PATTERN
∘ LOADED PATTERN
+ DIFFERENCE
y
P
C
O
x
D
4β
P
FRINGES
0
2
4
6
8
10
12
-1.00
-0.75
-0.50
-0.25
0
0.25
0.50
0.75
1.00
SECTION CD ψ/β

FIG. 7



P
A
A
O
x
4ρ
P
∘ PHOTOEL. POINTS
× INTERFER. POINTS
$(\sigma_1+\sigma_2)$/ FRINGES
1
2
3
4
5
6
7
-90°
-70°
-60°
-30°
-10°
0
10°
30°
50°
70°
90°
θ°

FIG. 8



y
P
A
B
O
x
4
P
FRINGES
7
6
5
4
3
2
1
0
0.25
0.50
0.75
1.00
x/β
PHOTOELASTIC POINTS
INTERFER. POINTS
FRINGES
5
4
3
2
1
0
0.25
0.50
0.75
1.00
ψ/β

FIG. 9

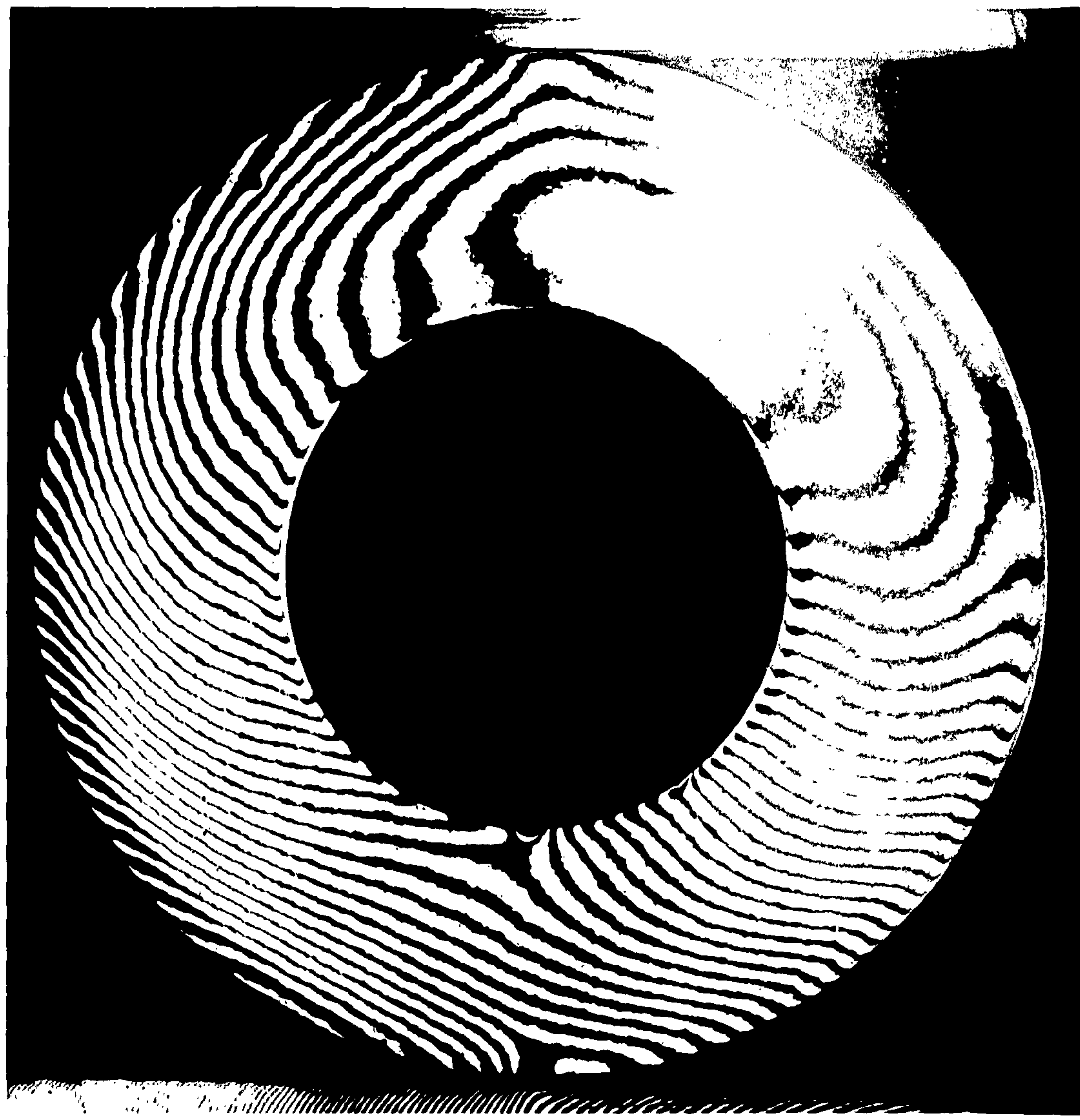


Fig 10(a)

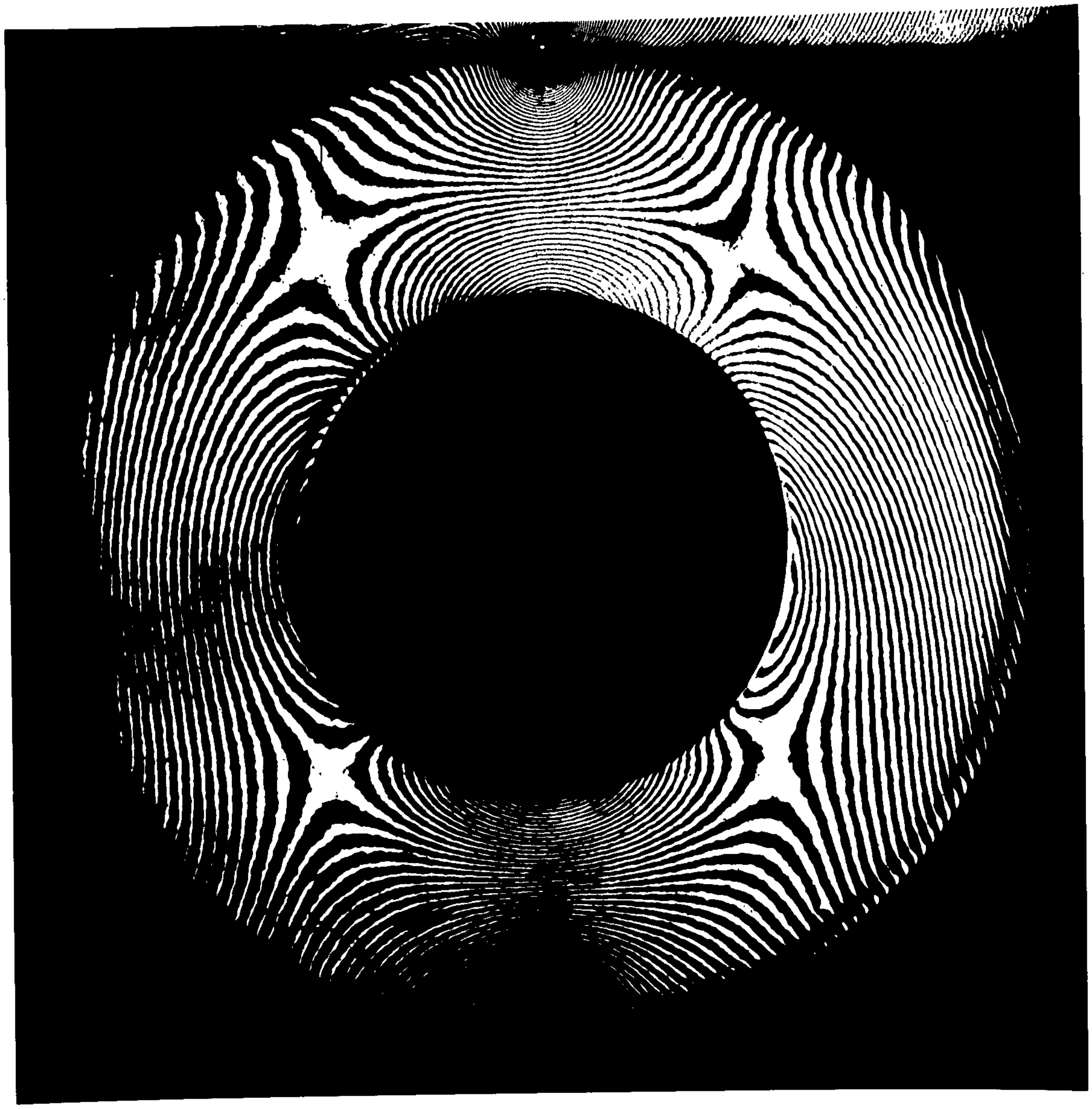
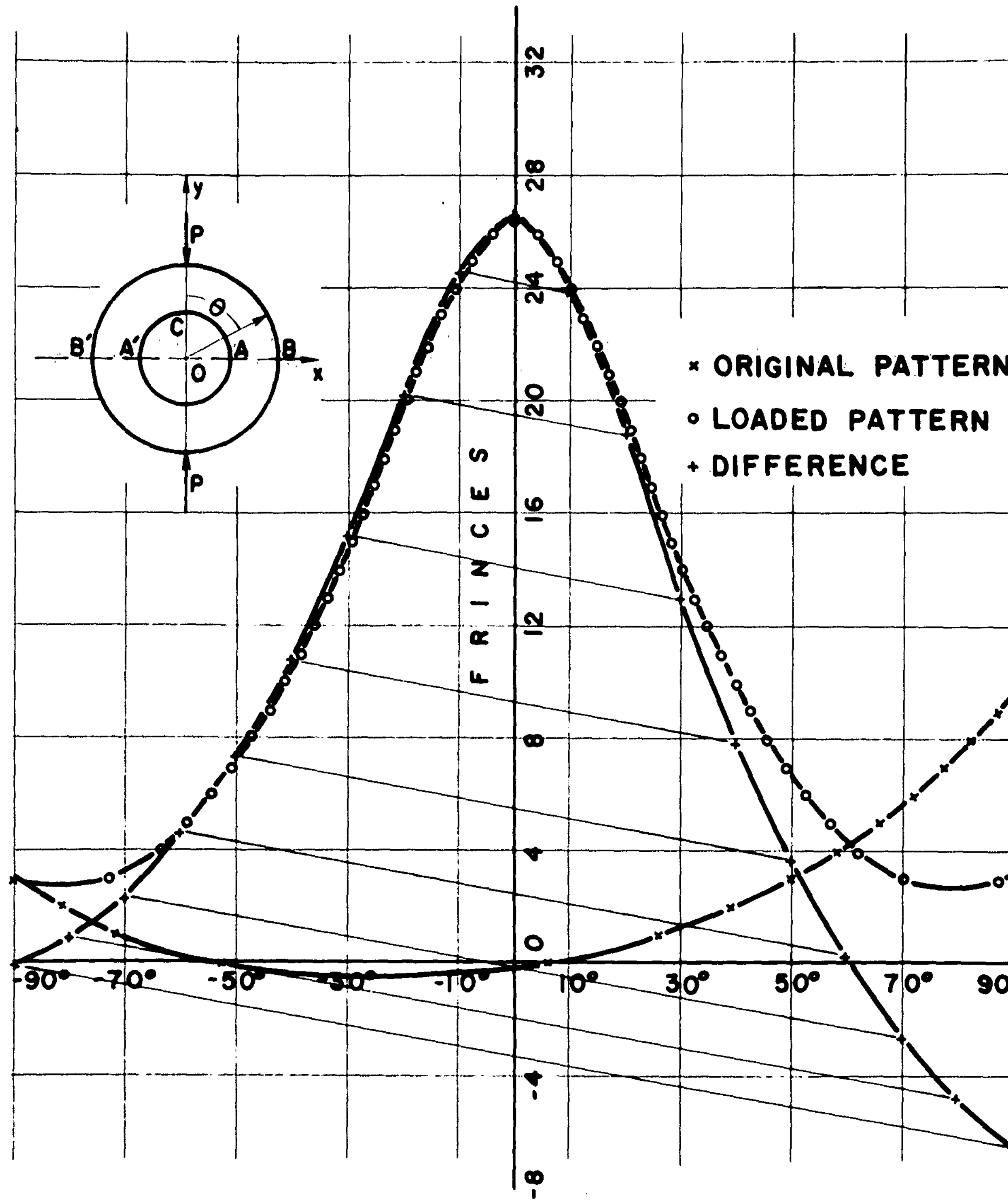
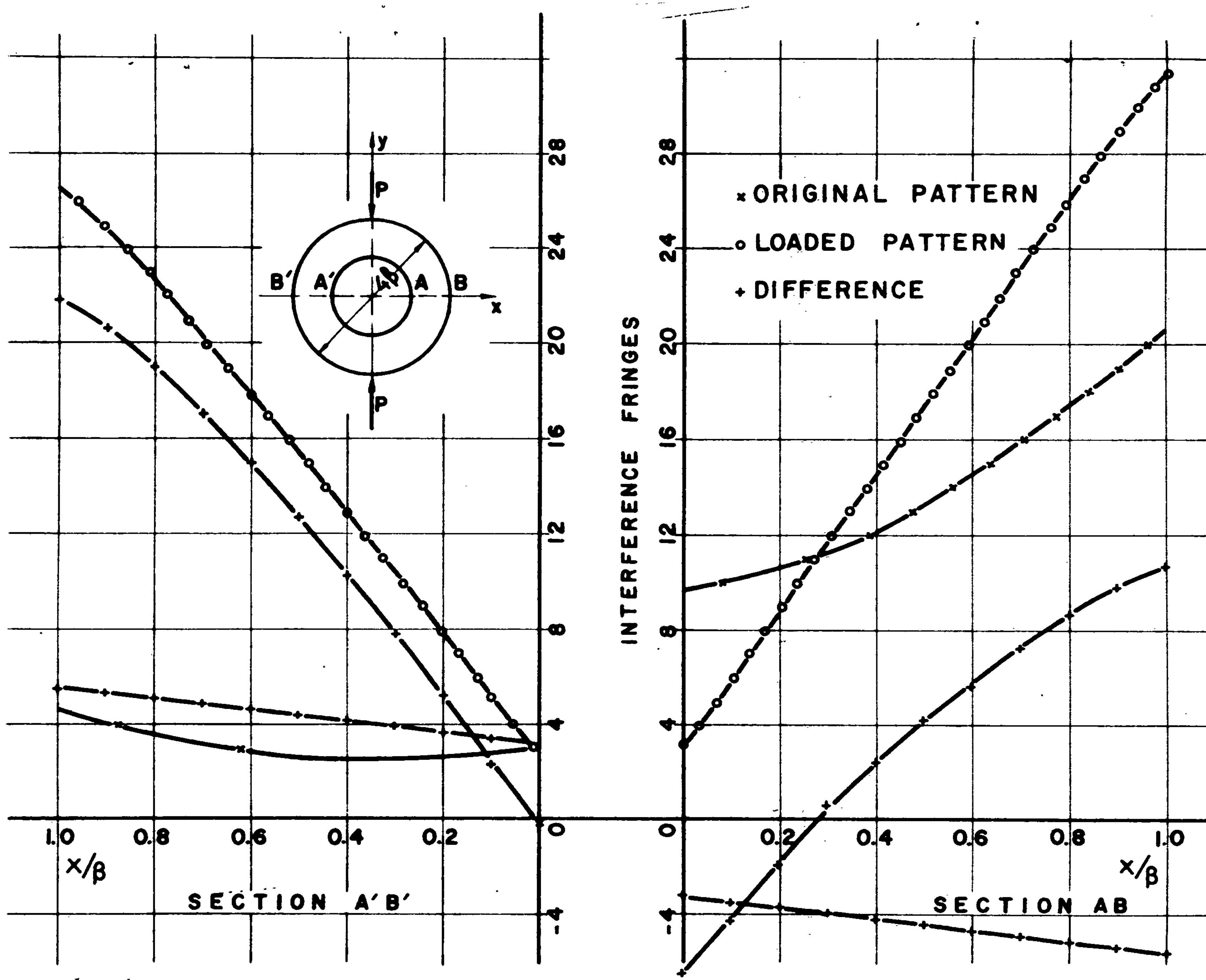


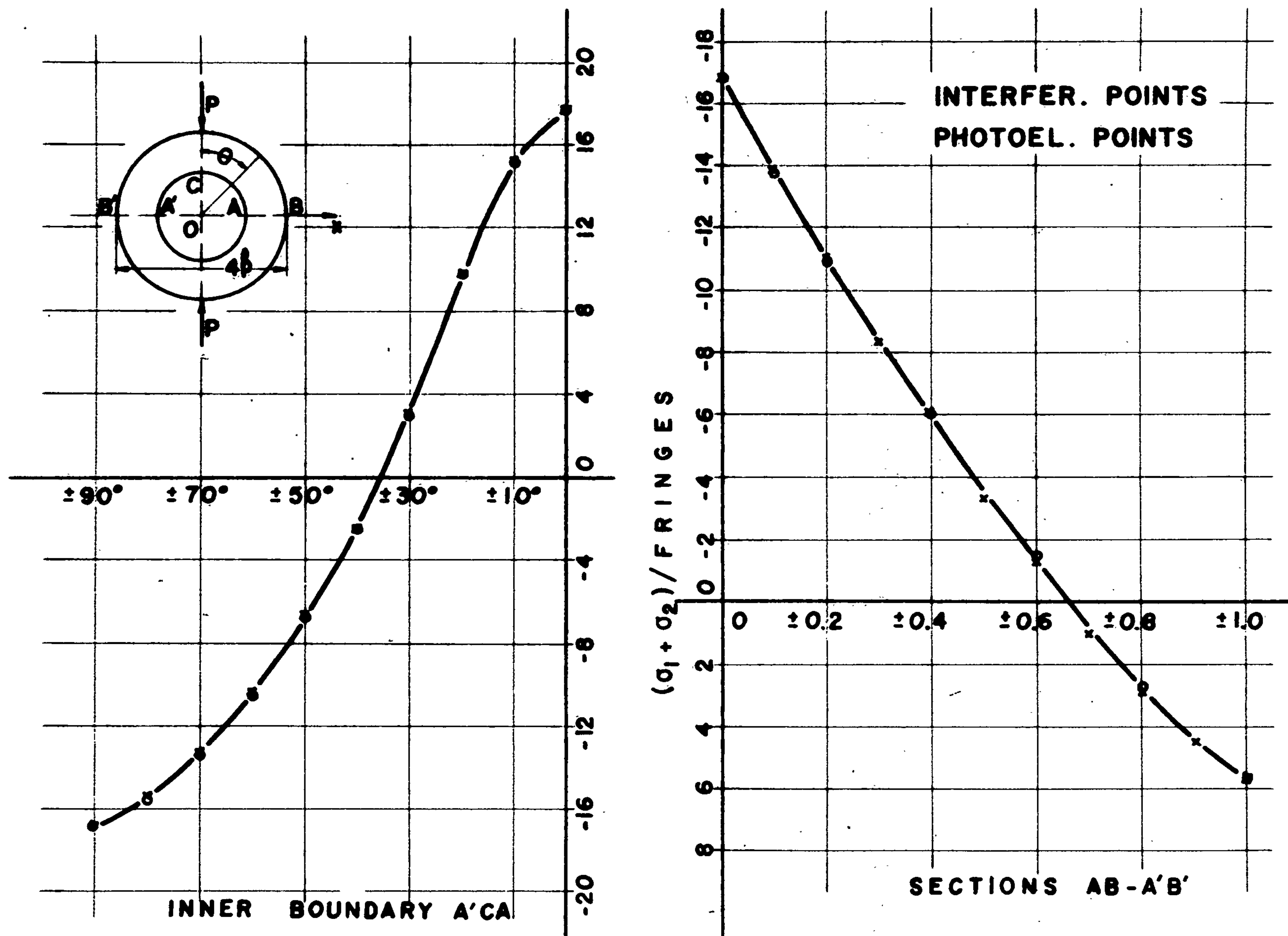
FIG. 10(B)



y
P
θ
C
B'
A'
A
B
x
O
P
× ORIGINAL PATTERN
○ LOADED PATTERN
+ DIFFERENCE
FRINGES
32
28
24
20
16
12
8
4
0
-4
-8
-90°
-70°
-50°
-30°
-10°
10°
30°
50°
70°
90°



× ORIGINAL PATTERN
○ LOADED PATTERN
+ DIFFERENCE
INTERFERENCE FRINGES
x/β
SECTION A'B'
SECTION AB
P
y
x
A
B
A'
B'


P
θ
C
B'
A'
A
B
x
O
4b
P
±90°
±70°
±50°
±30°
±10°
20
16
12
8
4
0
-4
-8
-12
-16
-20
INNER BOUNDARY A'CA
$(\sigma_1 + \sigma_2)$/FRINGES
-18
-16
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
0
±0.2
±0.4
±0.6
±0.8
±1.0
INTERFER. POINTS
PHOTOEL. POINTS
SECTIONS AB-A'B'

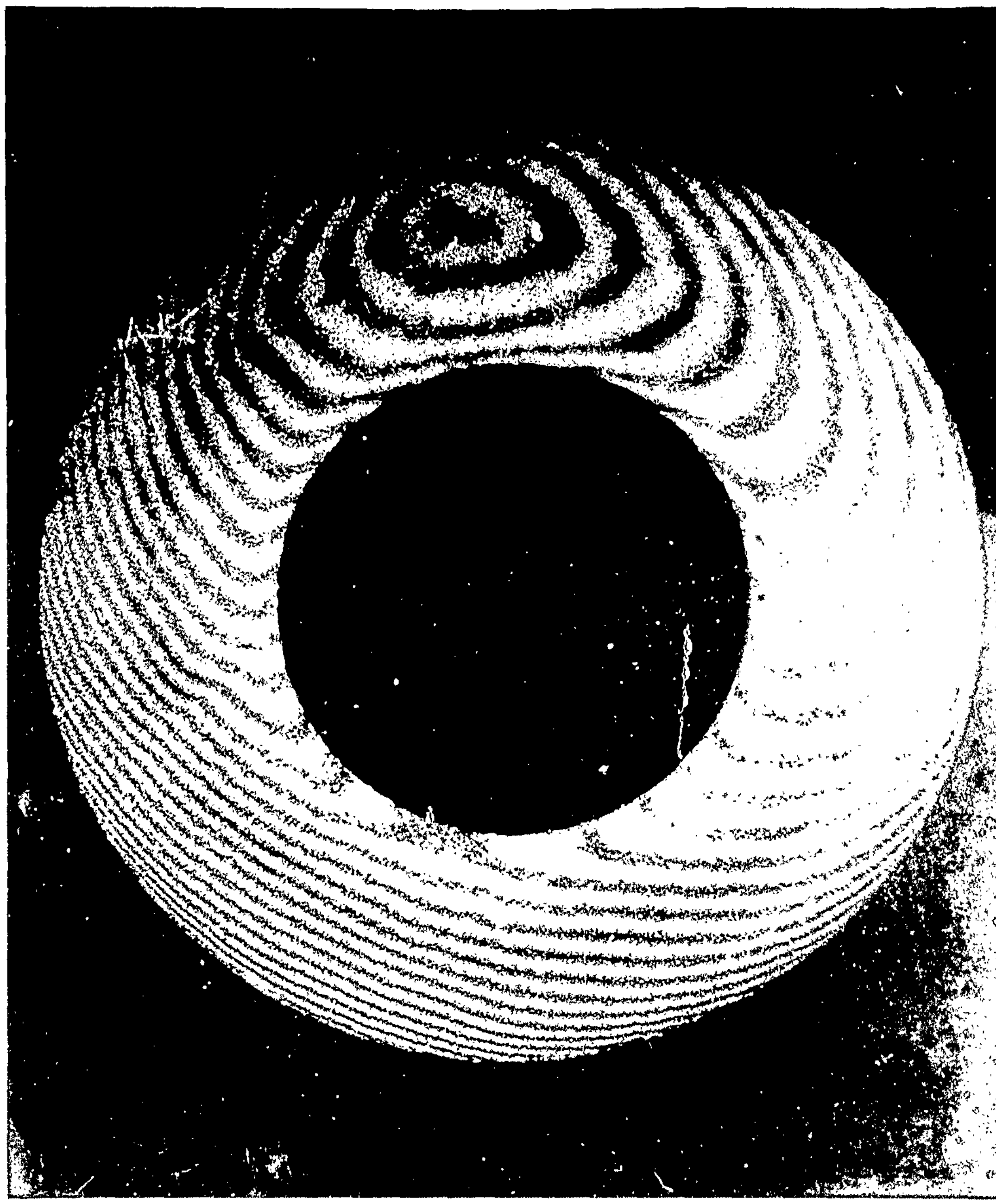


Fig. 14(a)

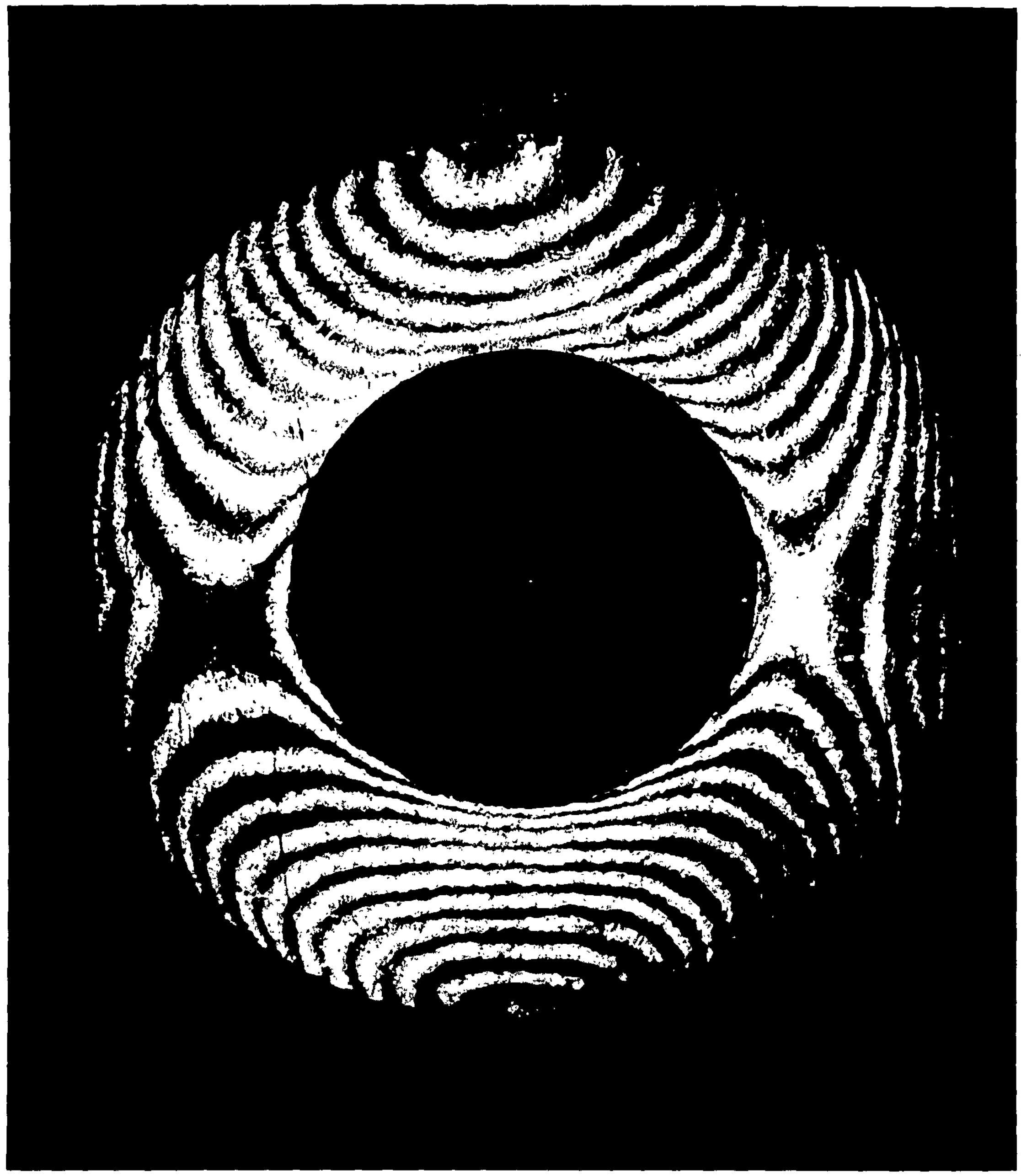


FiG 14(B)